Equation of the type: $pa^m + qb^m + rc^n + sd^n = 0$

$pa^m + qb^m + rc^n + sd^n$ =0 -----(1)

where: $(m, n) = (2,4)$

Case 1:

$$(p, q, r, s) = (1,1, -1, -1)$$

Substitution above values in equation (1) we get:

$$a^2 + b^2 = c^4 + d^4$$

Above is equivalent to:

$$(a + c^2)(a - c^2) = (d^2 + b)(d^2 - b)$$

Since, $(pq)(rs) = (ps)(rq)$, let:

$$(a + c^2) = pq$$

$$(a - c^2) = rs$$

$$(d^2 + b) = ps$$

$$(d^2 - b) = rq$$

Hence we get:

$$2c^2 = pq - rs$$

$$2d^2 = ps + qr$$

$$2a = pq + rs$$

$$2b = ps - qr$$

$$Let, \quad q = 2(p+r) \quad \& \quad s = 2p$$

Therefore we get: c=p, &

$d^2 = (p^2 + pr + r^2)$ -----(2)

Above equation (2) is parametrized as:

$$(p, r, d) = [(m^2 - n^2), (2mn + n^2), (m^2 + mn + n^2)]$$

Hence we get: $(c, d) = [(m^2 - n^2), (m^2 + mn + n^2)]$

Since, $2a = pq + rs$

we get, $a = (m^2 - n^2)(m^2 + 4mn + n^2)$

And as, $2b = ps - qr$

we get, $b = (m^4 - 2m^3n - 7m^2n^2 - 2mn^3 + n^4)$

for, $(m, n) = (3,2)$

$$(a, b, c, d) = (185,311,5,19)$$

we get the numerical solution:

$$(5^4 + 19^4) = (185^2 + 311^2)$$

Case 2

$$pa^m + qb^m + rc^n + sd^n = 0 \text{ -----(1)}$$

$$(p, q, r, s) = (5, -5, 1, -1)$$

$$(m, n) = (2,4)$$

Since we have,

$$5a^2 + c^4 = 5b^2 + d^4$$

Hence, $$5(a^2 - b^2) = c^4 - d^4$$

$$5(ab + a)(b - a) = (c^2 + d^2)(c^2 - d^2)$$

Hence,

$$c^2 - d^2 = b + a$$

$$c^2 + d^2 = 5b - 5a$$

we have, $c^2 = (3b - 2a)$ & $d^2 = (2b - 3a)$

Taking, $a = (10m^2 - 15n^2)$ & $b = (15m^2 - 10n^2)$

we get, $(c, d) = (5m, 5n)$

For, $(m, n) = (3,2)$ we get,

$$(a, b, c, d) = (30,95,15,10)$$

$$15^4 + 5(30)^2 = 10^4 + 5(95)^2$$

<u>Case 3</u>

The below identity was also derived:

$pa^m + qb^m + rc^n + sd^n = 0$ -------(1)

$$(p, q, r, s) = (6, 1, -9, -1)$$

$$(m,n)=(2,4)$$

Hence equation (1) becomes:

$$6a^2+b^2=9c^4+d^4\ ----(3)$$

equation (3) has solution:

$$(a,b,c,d)=[(2t^2+6t+3),(t^2+t),((t+1),(t)]$$

For t=6, we get:

$$(a,b,c,d)=(7,6,42,111)$$

$$6(42)^2+(111)^2=9(7)^4+(6)^4$$

After removing common factor's we get:

$$6(14)^2+(37)^2=9(2)^4+(7)^4$$
