Various identities

Second Powers:

$$(m^2)(a^2+b^2)=(n^2)(c^2+d^2)$$

$$a=u^2-v^2$$

$$b=2uv$$

$$c=p^2-q^2$$

$$d=2pq$$

$$m=p^2+q^2$$

$$n=u^2+v^2$$

$$For\ (u,v)=(3,2)\ \&\ (p,q)=(4,1)\ we\ get:$$

$$289(12^2+5^2)=169(15^2+8^2)$$

Third powers:

$$(a^3+b^3=c^3+d^3)$$

We have the identity:

$$(m^2-np)^3+(nq-m^2)^3=(mq-n^2)^3+(n^2-mp)^3$$

With the condition:

$$(p^2+pq+q^2)=(3mn)\ -----(1)$$

Parametrizing equation (1) at (p,q,m,n)= (1,1,1,1) we get:

$$(p, q, m, n) =$$

$$\left((k^2 + 8k - 5), (-5k^2 + 8k + 1), (k^2 - 4k + 7), (7k^2 - 4k + 1)\right)$$

For k=2: we get,

$(m, n, p, q) = (3,21,15, -3)$ and numerical solution after taking out common factors:

$$(17,4)^3 = (25, -22)^3$$

Third powers:

We have identity:

$$(a + b + c)^3 + (a - b - c)^3 + (-a + b - c)^3 + (-a - b + c)^3 = 24abc$$

Parametrizing, $(24abc) = (w^3)$ $\quad at\ (a, b, c) = (9,4,2)\ \&\ w = 3b$

We get:

$$a = 25$$

$$b = 20(2k - 1)$$

$$c = 18(2k - 1)^2$$

$$w = 60(2k - 1)$$

At , k=1 we get (a,b,c,w)=(25,20,18,60)

Numerical solution is:

$$(63)^3 + (-13)^3 + (-23)^3 + (-27)^3 = 60^3$$

Fourth powers:

We have Identity by Lucas:

$$\big((a+b),(a-b),(b+c),(b-c),(c+a),(c-a)\big)^4 + 2(a,b,c)^4 = 6(a^2+b^2+c^2)^2$$

We have Identity by Lucas:

$$\big((a+b),(a-b),(b+c),(b-c),(c+a),(c-a)\big)^4 + 2(a,b,c)^4 = 6(a^2+b^2+c^2)^2$$

Since $(a^2+b^2+c^2) = (n^2+n+1)^2$

$$for\ (a,b,c) = (n, n+1, n^2+n)$$

Hence we get:

$$\big(1,(2n+1),(n^2+2n+1),(n^2-1),(n^2+2n),(n^2)\big)^4 + 2(n,n+1,n^2+n)^4 = 6(n^2+n+1)^4$$

$$For\ n = 3\ we\ get:$$

$$(1,7,16,8,15,9)^4 + 2(3,4,12)^4 = 6(13)^4$$

Fifth Powers:

If, $(ab+bc+ca) = 0$ & $(a+b) = (c+d)$ and,

$m(a^5+b^5+c^5+(a+b+c)^5+(-a-b)^5+(-b-c)^5+(-c-a)^5) = n(d^5+e^5+f^5+(d+e+f)^5+(-d-e)^5+(-e-f)^5+(-f-d)^5)$

Where:

$$m = (ab)^2(a^2 + ab + b^2)^2$$

$$n = (de)^2(d^2 + de + e^2)^2 \quad \text{then,}$$

$$For\ (a,b,c) = (15,10,-6)\ \&\ (d,e,f) = (20,5,-4), \quad we\ get\ after\ removing\ common\ factors:$$

$$m(15,10,-6,19,-25,-4,-9)^5 = n(20,5,-4,21,-25,-16,-1)^5$$

$$Where\ m = 70\ \&\ n = 471$$

Sixth powers:

If, $(ab + bc + ca) = 0$ & $(a + b) = (c + d)$ and,

$$m = (ab)^2(a^2 + ab + b^2)^3$$

$$n = (de)^2(d^2 + de + e^2)^3 \quad \text{than,}$$

$$m(d^6 + e^6 + f^6 + (d + e + f)^6) + n((a + b)^6 + (b + c)^6 + (c + a)^6) =$$

$$m((d + e)^6 + (e + f)^6 + (f + d)^6) + n(a^6 + b^6 + c^6 + (a + b + c)^6)$$

$$For\ (a,b,c) = (15,10,-6)\ \&$$

$$(d,e,f) = (20,5,-4), \quad we\ get\ after\ removing\ common\ factors:$$

$$m(21,20,5,4)^6 + n(25,4,9)^6 = m(25,16,1)^6 + n(15,10,19,6)^6$$

$$Where\ m = 6859\ \&\ n = 4116$$

Combination of 2nd, 3rd & 6th powers:

$$(a^6 + b^6 + c^6) = 2(-ab - bc - ca)^3 + 3(abc)^2$$

$$If\ (a + b + c) = 0$$

$$For\ (a, b, c) = \ (3, -2, -1)\ \ we\ get:$$

$$(3,2,1)^6 = \ 2(7)^3 + 3(6)^2$$
