Parametric solutions to the below mentioned equation

Equation: $$pa^n + qb^n = rc^n$$

For n=2,3,4,5 & 6

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n=2

$$pa^2 + qb^2 = rc^2$$

$(a,b,c) =$

$$\left((5k^2 - 20k - 4), (10k^2 + 20k - 32), (25k^2 - 20k + 60)\right)$$

$(p,q,r) = (35,10,3)$

$35(19)^2 + 10(2)^2 = 3(65)^2$

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n=3

$$pa^3 + qb^3 = rc^3$$

$(a,b,c) = \left((m-1),(m),(m+1)\right)$

(p,q,r) = $\left((m^3 + 6m^2 + 6m + 2), (2m^3 + 6m), (3m^3 - 6m^2 + 6m - 2)\right)$

For value m=6

$235(5)^3 + 234(6)^3 = 233(7)^3$

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n=4

$$pa^4 + qb^4 = rc^4$$

p=(a^2)

q=(2b^2-3a^2)

r=(a^2+2b^2), condition is; $c^2 = b^2 - a^2$

For (a,b,c)=(4,5,3) we get,

(p,q,r,a,b,c)=(16,2,66,4,5,3)

125(3)^4+29(10)^4 = 125(7)^4

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n=5

$$pa^5 + qb^5 = rc^5$$

Let (p,q,r)=(w+1,w,w-1) and (a,b,c)=(m-1.m.m+1)

After simplification we get:

p=($m^5 + 20m^3 + 10m^4 + 20m^2 + 10m + 2$)

q=$2m(m^4 + 10m^2 + 5)$

r= ($3m^5 + 20m^3 - 10m^4 - 20m^2 + 10m - 2$)

For m=2 we get $(p, q, r, a, b, c) = (227,122,17,1,2,3)$

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n=6

$$pa^6 + qb^6 = rc^6$$

Has solution: $p = a^4 + 3a^2b^2 + 4b^4$

$$q = 4a^4 + 3a^2b^2 + b^4$$

$$r = a^4 + b^4$$

$$Condition: \quad c^2 = a^2 + b^2$$

$Numerical\ solution\ for\ (a, b, c) = (4,3,5)\ is \quad (p, q, r, a, b, c) =$ (1012,1537,337,4,3,5)

<u>Table:</u>

For n=2,3,4,5 & 6

n=2 (p,q,r,a,b,c) = (7,2,25,19,2,13)
n=3 (p,q,r,a,b,c) = (235,234,233,5,6,7)
n=4 (p,q,r,a,b,c) = (125,29,125,3,10,7)
n=5 (p,q,r,a,b,c) = (227,122,1,2,5,3)
n=6 (p,q,r,a,b,c) = (1012,1537,337,4,3,5)
