Six variables used for taxicab equation having three chains:

$$Taxicab\ equation\ for\ k-2-3-3-3\ \ for\ k\ =\ 2\ \&\ 4$$

$$We\ have\ the\ well\ known\ \ Identity:$$

$$(a^2+2ac-2bc-b^2)^k+(b^2-2ab-2ac-c^2)^k+(c^2+2ab+2bc-a^2)^k$$
$$=2(a^2-ab+ac+bc+b^2+c^2)^k$$

$$Substituting\ a=d, b=e\ \&\ c=f\ we\ get:$$

$$(d^2+2df-2ef-e^2)^k+(e^2-2de-2df-f^2)^k+(f^2+2de+2ef-d^2)^k=$$
$$2(d^2-de+e^2+df+ef+f^2)^k$$

After some algebra similar to used for taxicab equation for four variables in my previous article we get:

$$2(d^2-de+e^2+df+ef+f^2)^k*(a^2-ab+b^2+ac+bc+c^2)^k=$$

$$(d^2+2df-2ef-e^2)^k*(a^2-ab+b^2+ac+bc+c^2)^k+$$
$$(e^2-2de-2df-f^2)^k*(a^2-ab+b^2+ac+bc+c^2)^k+$$
$$(f^2+2de+2ef-d^2)^k*(a^2-ab+b^2+ac+bc+c^2)^k=$$

$$(d^2-de+e^2+df+ef+f^2)^k*(a^2+2ac-2bc-b^2)^k+$$
$$(d^2-de+e^2+df+ef+f^2)^k*(c^2+2ab+2bc-a^2)^k+$$
$$(d^2-de+e^2+df+ef+f^2)^k*(b^2-2ab-2ac-c^2)^k=$$

$$(cd+bd+ae-be+af+cf)^k*(bd-2ad-cd+ae+be+2ce-af+2bf+cf)^k+$$
$$(cf+bf-ad+bd+ae+ce)^k*(bf-2af-cf-ad-bd-2cd-ae+2be+ce)^k+$$
$$(-be+2ae+ce-ad+2bd+cd+af+bf+2cf)^k*(be+ce-ad-cd-af+bf)^k=$$

$$(-bf-cf-ad-cd+ae-be)^k*$$
$$(bf-2af-cf-ad+2bd+cd-ae-be-2ce)^k+$$
$$(-be+2ae+ce-ad-bd-2cd+af-2bf-cf)^k*$$
$$(-c*e-b*e+b*d-a*d-a*f-c*f)^k+$$
$$(bd-2ad-cd+ae-2be-ce-af-bf-2cf)^k*$$
$$(-cd-bd+ae+ce-bf+af)^k$$

For $a=2, b=1, c=1, d=3, e=2, f=1$ *we get the numerical solution*

after removing the common factors:

$$2(91)^k=(105{,}49{,}56)^k=(104{,}65{,}39)^k=(96{,}85{,}11)^k=(99{,}80{,}19)^k$$
