VARIOUS DERIVED IDENTITIES;

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Degree Two:

$$2-2-2-2\ eqn$$

$$2*(25x^2+25y^2)^2 =$$

$$(-17y^2+17x^2-62x*y)^2+\ (-31y^2+31x^2+34xy)^2 =$$

$$(-5y^2+5x^2-70xy)^2+\ (-35y^2+35x^2+10xy)^2.$$

$$For\ x=3, y=2\ we\ get,$$
$$2*(325)^2 =\ (287{,}359)^2 =\ (235{,}395)^2$$

Degree Three:

$$\boldsymbol{Equation}\quad (\boldsymbol{p})^3 =\ (\boldsymbol{a},\boldsymbol{b},\boldsymbol{c})^3 =\ (\boldsymbol{d},\boldsymbol{e},\boldsymbol{f})^3$$

$$Below\ mentioned\ Parametric\ solution\ worked\ out\ by\ me;$$

$$(21808)^3*((61k^2+4k-38k^3+3+42k^4)^2)^3 =$$
$$((2k+1+21k^2)^2)^3$$
$$*((728272k^2-367600k-39840k^3-133784$$
$$-\quad 93664k^4)^3\ +$$
$$(172336k^2-615696k+372640k^3+67224-100640k^4)^3+$$
$$(123776k^2-15872k-257664k^3+212912+135808k^4)^3)\ =$$

$$((-2k+3+2k^2)^2)^3 * ((1513672k^2 - 292496k + 7695408k^3 - 24516 - 3892644k^4)^3 + (-1279496k^2 - 431600k + 6892368k^3 - 5660 + 5481924k^4)^3 + (944624k^2 + 268256k - 1815072k^3 + 29352 + 9220008k^4)^3) .$$

$For\ k = 2, \quad we\ get\ after\ removing\ common\ factors:$

$All\ elements\ (p, a, b, c, d, e, f) are\ all\ positive\ integers$

$$(7^2 * 89^2)^3 * (5452)^3 = (7^2)^3 * (1181535^3 + 34335978^3 + 34215721^3) = (89^2)^3 * (56690^3 + 224014^3 + 196972^3)$$

Degree Five:

$$2 * 49^5 * (3x^2 + 9y^2)^5 = (17x^2 + 243y^2 + 376xy)^5 + (81x^2 + 51y^2 - 376xy)^5 + (-29x^2 + 381y^2 - 284xy)^5 + (127x^2 - 87y^2 + 284xy)^5 + (-61x^2 + 477y^2 + 92xy)^5 + (159x^2 - 183y^2 - 92xy)^5$$

We also Have for degree five:

$$2 * (3x^2 + 9y^2)^5 =$$

$$(-x^2 + 9y^2 + 4xy)^5 + (-x^2 + 9y^2 - 4xy)^5 + (3x^2 - 3y^2 + 4xy)^5 + (3x^2 - 3y^2 - 4xy)^5 + (x^2 + 3y^2 + 8xy)^5 + (3x^2 + 3y^2 - 8xy)^5$$

Degree Two & Four:

$$\mathbf{2nd\ degree}$$

$$(2-2-3-3)eqn$$

$$2(w)^2 = p^2+q^2+r^2 = s^2+t^2+u^2$$

$$\begin{aligned}2(w)^2 &= \left(48v(u+2v)\right)^2+\left(3(u^2+4u*v-28v^2)\right)^2 \\ &+\left(3(u^2+4u*v-28v^2)\right)^2 \\ &= (4u^2-144v^2)^2+(u^2+36u*v+36v^2)^2 \\ &+(u^2+36uv+36v^2)^2\end{aligned}$$

$$where$$

$$a = 4u^2-144v^2$$

$$b = u^2+36uv+36v^2$$

$$c = -(2u^2+24uv-24v^2)$$

$$w = 3(u^2+4uv+36v^2)$$

$$\mathbf{3rd\ degree}, \boldsymbol{k=3}$$

$$2(w)^3 = p^3+q^3+r^3 = s^3+t^3+u^3$$

$$(p,q,r,s,t,u)takes\ on\ signed\ values$$

$$3-2-3-3\ eqn$$

$$\begin{aligned}2(4a^3b^3)^3 &= (4a^3b^3+24c^3)^3+(4a^3b^3-24c^3)^3+(-24abc^2)^3 \\ &= (4a^3b^3+3c^3)^3+(4a^3b^3-3c^3)^3+(-6abc^2)^3\end{aligned}$$

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4 th degree

$$2*(169)^4*[(3*q^2+3*q*p+p^2)^4]=$$
$$=169\text{^}4*(p^4*(p+2*q)^4+(p+q)^4*(p+3*q)^4+d^4(2p+3q)^4)$$
$$=[(24*q+23*p)\text{^}4*(22*q+7*p)\text{^}4+(q+8*p)\text{^}4*(45*q+22*p)\text{^}4+$$
$$(23*q+15*p)^4*(-21*q+p)^4]$$

Where:

$$p=19(2k-1)$$
$$q=13k^2-18k-4$$

For $k=1$ *we get, after dividing by* $(169)^4$

$$2*(91)^4=(19,80,99)^4=(85,11,96)^4$$

$$4-2-3-3\ eqn$$

$$2(w)^4=p^4+q^4+r^4=s^4+t^4+u^4$$

$$2(w)^4=(a^2-b^2)^4+(2ab-b^2)^4+(2ab-a^2)^4$$

$$=[19c(19c+2d)]^4+[38*cd+3d^2]^4+[(19c+d)(19c+3d)]^4$$

Where:

$$a=23k^2+12k-29$$
$$b=k^2+22k-12$$
$$c=2k-1$$
$$d=13k^2-18k-4$$

$$w=(a^2-ab+b^2)=(3d^2+57cd+19^2)$$
$$w=13(39k^4+6k^3-53k^2-34k+49)$$

Consider degree four eqn. below:

$$2(w)^4 = (a, b, c)^4 = (d, e, f)^4$$

This is paramatised as below

$$12^4[(u^2 - v^2)^4 + (u^2 + 2uv)^4 + (v^2 + 2uv)^4] =$$

$$(4(u + v)m)^4 + (u + v + m)^4(m - 3u - 3v)^4 +$$

$$(u + v - m)^4(3u + 3v + m)^4$$

$= 2(12)^4(u^2 + uv + v^2)^4$

For u=6,v=5 m=27 we get: after taking out common factors

$$2(91)^4 = (11, 96, 85)\hat{}4 = (19, 80, 89)^4$$

Also:

$2(m^2 + 3v^2 + 6uv + 3u^2)^4 =$

$$(4(u + v)m)^4 + (u + v + m)^4(m - 3u - 3v)^4 +$$

$$(u + v - m)^4(3u + 3v + m)^4$$

where $m^2 = (9u^2 + 6uv + 9v^2)$

$$\text{The above has solution as } u = 6, v = 5 \text{ and } m = 27$$

$$\text{Thus it can be paramatised as below}$$

$$u = \frac{2(-14-15k+9k^2)}{3+2k+3k^2}$$

$$v = -\frac{3(-5+12k+9k^2)}{3+2k+3k^2}$$

$$\text{Thus we get infinite values of } (u, v)$$

$$Example$$

$$For\ k = 2\ we\ get:$$

$$2(30121)^4 =$$

$$(26969, 5536, 32505)^4 = (29216, 30951, 1735)^4$$

3rd degree, k=3

$2(w)^3 = p^3 + q^3 + r^3 = s^3 + t^3 + u^3$

$$(p, q, r, s, t, u) takes\ on\ signed\ values$$

$2(4a^3b^3)^3 = (4a^3b^3 + 24c^3)^3 + (4a^3b^3 - 24c^3)^3 + (-24abc^2)^3$

$$= (4a^3b^3 + 3c^3)^3 + (4a^3b^3 - 3c^3)^3 + (-6abc^2)^3$$

$$= (4a^3b^3 + 81c^3)^3 + (4a^3b^3 - 81c^3)^3 + (-54abc^2)^3$$

$$for\ a = b = 1\ and\ c = 2, we\ get$$

$$2(4)^3 = (196 - 188, -96)^3 = (28, -20, -24)^3 = (652, -644, -216)^3$$
